

# How ‘Backward Looking’ is the New Keynesian Phillips Curve?

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## **Abstract**

In this paper I develop an empirical test to disentangle a “fully forward looking” New Keynesian Phillips curve from a partially “backward looking” or “hybrid” specification for the dynamic time series representation of inflation. The test is based on impulse response shapes which can be estimated by “local projections” without specifying a particular time series model for the dynamic path of inflation. I specify a null hypothesis in line with impulse response functions implied by state of the art sticky price DSGE models á la Christiano, Eichenbaum & Evans (2005) or Smets & Wouters (2003) which incorporate both the “forward looking” and the “hybrid” specification as special cases. The properties of the test are illustrated in Monte Carlo experiments.

## **1 Introduction**

In the recent macroeconomic literature there is an ongoing debate about the “correct” specification of an empirical model for inflation dynamics (Gali & Gertler 1999, Gali, Gertler & Lopez-Salido 2005, Linde 2005, Rudd & Whelan 2005). In particular, there is no consensus on whether the path of inflation is best described by the so-called New Keynesian Phillips curve, a purely forward looking rational expectations approach as supported by Gali & Gertler (1999), or whether one

should incorporate lagged or “backward looking” terms in the specification of the dynamic inflation equation. The latter approach can be rationalized by recent sticky price DSGE models in the style of Christiano et al. (2005) or Smets & Wouters (2003), who augment the well known Calvo (1983) pricing model by the assumption that firms who cannot re-optimize at a given point in time index the price of their product to the most recent measure of inflation.

In this paper I construct a test based on impulse response shapes which is able to disentangle the two opposing models. I make use of the fact that each empirical specification mentioned above can be interpreted as a special case of the theoretic framework developed in, e.g., Smets & Wouters (2003). This allows me to analyze the theoretic predictions of either specification within the same theoretic model and identify the differences in the dynamics implied by the two opposing variants of the model. Based on these differences in the implied dynamics, which can be summarized by impulse response functions, I set up a Wald test, using the methods developed by Jordà (2007), which allows me to empirically disentangle the two opposing model specifications. I illustrate the properties of my test in Monte Carlo experiments which show that the test has sufficient power to disentangle a “hybrid” calibration that corresponds to the estimates of Linde (2005) from a fully forward looking specification, which Gali & Gertler (1999) consider a reasonable approximation to the observed U.S. inflation dynamics. One of the virtues of this test is that it does not rely on the particular method used to generate the empirical impulse response functions. Hence, if one wants to be agnostic about the underlying empirical model for the dynamics of the system of endogenous variables one can use “model free” local projections, as suggested by Jorda (2005), in order to estimate impulse response functions and test for the preferred theoretic model specification.

The paper is organized as follows: Section 2 outlines the theoretic model, while

sections 3 and 4 discuss its solution and implied theoretic predictions for the dynamics of inflation. In section 5 I construct the test and illustrate its properties using Monte Carlo simulations. Finally, section 6 concludes.

## 2 The Model

To address the question motivated above I analyze the structural model as specified in Linde (2005) which corresponds to the linearized version of Euler equations derived from New Keynesian DSGE models in the line of Christiano et al. (2005) or Smets & Wouters (2003) and can be written as:

$$\pi_t = \omega_f E_t \pi_{t+1} + \omega_b \pi_{t-1} + \omega_y y_t + \epsilon_{\pi,t} \quad (1)$$

$$y_t = \beta_f E_t y_{t+1} + \beta_b y_{t-1} - \beta_r (r_t - E_t \pi_{t+1}) + \epsilon_{y,t} \quad (2)$$

$$r_t = (1 - \rho)(\gamma_\pi \pi_t + \gamma_y y_t) + \rho r_{t-1} + \epsilon_{r,t}, \quad (3)$$

where I assume that  $(\epsilon_{\pi,t}, \epsilon_{y,t}, \epsilon_{r,t})' \sim N(0, \Sigma_\epsilon)$ , with  $\Sigma_\epsilon$  diagonal,  $\pi_t$  measures the rate of inflation,  $y_t$  denotes the output-gap,  $r_t$  captures percentage changes in the nominal interest rate, and the vector  $\theta = (\omega_f, \omega_b, \omega_y, \beta_f, \beta_b, \beta_r, \gamma_\pi, \gamma_y, \rho)'$  is composed of structural parameters from the underlying theoretic model. In the context of this model the question motivated in the previous section corresponds to asking whether, according to the data, the aggregate supply relation (1) has a backward looking term, i.e.  $\omega_b \neq 0$ . To understand the essence of that question it is useful to investigate the derivation of equation (1). In a standard New Keynesian DSGE framework a relation of the form (1) arises as a consequence of Calvo (1983) style price stickiness together with the assumption of inflation indexation. For the particular question raised above the latter is crucial and deserves some explanation

here. In the standard Calvo (1983) setup an intermediate firm  $i$  cannot reset its price with fixed probability  $\eta \in (0, 1)$  each period and hence keeps last period's price  $p_t(i) = p_{t-1}(i)$ . Instead, one can assume that firms which cannot update their price in period  $t$  follow the inflation indexation rule

$$\log p_t(i) = \log p_{t-1}(i) + \gamma\pi_{t-1}, \quad (4)$$

when setting their price, where the parameter  $\gamma \in [0, \infty)$  captures the degree of indexation to the most recent inflation measure  $\pi_{t-1}$ . Given the above indexation rule, the linearized optimality condition for the firms' optimal price setting problem becomes

$$\pi_t - \gamma\pi_{t-1} = \kappa y_t + \beta E_t(\pi_{t+1} - \gamma\pi_t), \quad (5)$$

where  $\beta \in (0, 1]$  is the households' subjective discount factor and

$$\kappa \equiv \frac{(1 - \eta)(1 - \eta\beta)}{\eta} \xi,$$

where  $\xi > 0$  is a constant containing other structural parameters, which are not of particular interest here. Rewriting (5) yields

$$\pi_t = \frac{\beta}{1 + \beta\gamma} E_t \pi_{t+1} + \frac{\gamma}{1 + \beta\gamma} \pi_{t-1} + \frac{\kappa}{1 + \beta\gamma} y_t, \quad (6)$$

which directly implies that the parameters  $(\omega_f, \omega_b, \omega_y)$  in equation (1) implicitly have to satisfy the restrictions

$$\omega_f = \frac{\beta}{1 + \beta\gamma} \quad (7)$$

$$\omega_b = \frac{\gamma}{1 + \beta\gamma} \quad (8)$$

$$\omega_y = \frac{(1 - \eta)(1 - \eta\beta)}{\eta(1 + \beta\gamma)} \xi. \quad (9)$$

Notice that these restrictions contain two interesting special cases: First, if  $\gamma = 1$ , as assumed in Christiano et al. (2005), the above restrictions imply that  $\omega_f + \omega_b = 1$  and even for values of  $\beta$  close to 1 we have that  $\omega_b > \omega_f$ . Both implications seem to be at odds with the NLS estimates of  $\omega_f$  and  $\omega_b$  as reported in Linde (2005). While Linde (2005) finds FIML estimates that are in line with  $\omega_f$  and  $\omega_b$  summing up to unity, his estimate  $\hat{\omega}_f \approx 0.3$  implies an unreasonably low value for  $\beta \approx 0.42$ , given that he implicitly assumes  $\gamma = 1$ . In that sense, his estimates are not 'literally' consistent with the theory either.

Second, a value of  $\gamma = 0$  implies that  $\omega_f = \beta$ ,  $\omega_b = 0$ , and  $\omega_y = \kappa$ , which is the case where the aggregate supply relation (1) reduces to the so-called New Keynesian Phillips curve. This special case, however, implies that for the calibration considered in Linde (2005) there is no stable solution to the system (1) – (3) that satisfies restrictions (7) through (9), when  $\gamma = 0$  exactly. Consequently, in order to study the dynamics of this special case, one has to rely on an approximation where  $\gamma \approx 0$  which gives rise to a stable solution.

In this paper I try to disentangle these two special cases (“hybrid” versus “fully forward looking”) with the use of structural impulse analysis. Following Jordà (2007), I construct a Wald test and analyze its properties with the use of Monte Carlo experiments based on OLS estimates of the VAR(1) solution to equations (1)

– (3). Even though this test does not solve the problem of how to consistently estimate the parameter vector  $\theta$  and  $\Sigma_\epsilon$  and conduct meaningful inference on these parameter estimates, the information uncovered is a useful guideline for theoretic considerations. In particular, it evaluates whether inflation indexation is a reasonable assumption or not. In the next two sections I investigate the theoretic properties of the system (1) – (3) and derive cross parameter restrictions, which allow me to fully identify all structural parameters in equations (1) – (3).

### 3 Solution and Structural Identification

In order to set up a Monte Carlo experiment to illustrate my test I first need to analyze the solution and implied cross-parameter restrictions of the model introduced in the previous section. The matrix version of the linear system (1) – (3) can be written as

$$\begin{bmatrix} 1 & -\omega_y & 0 \\ 0 & 1 & \beta_r \\ (\rho - 1)\gamma_\pi & (\rho - 1)\gamma_y & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ r_t \end{bmatrix} = \begin{bmatrix} \omega_f & 0 & 0 \\ \beta_r & \beta_f & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t \pi_{t+1} \\ E_t y_{t+1} \\ E_t r_{t+1} \end{bmatrix} + \begin{bmatrix} \omega_b & 0 & 0 \\ 0 & \beta_b & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\pi,t} \\ \epsilon_{y,t} \\ \epsilon_{r,t} \end{bmatrix}, \quad (10)$$

or shorthand

$$C_0 z_t = C_1 E_t z_{t+1} + C_2 z_{t-1} + \epsilon_t. \quad (11)$$

Christiano (2002) shows that rational expectations models of the form (11) have a VAR(1) solution given by

$$z_t = A z_{t-1} + B \epsilon_t, \quad (12)$$

where the coefficient matrices  $A$  and  $B$  have to satisfy the following restrictions

$$C_2 = C_0 A - C_1 A^2 \quad (13)$$

$$B = I_3 + C_0^{-1} C_1 A. \quad (14)$$

Given a solution for  $A$  the restrictions implied by (13) are sufficient to derive all structural parameters in  $C_0$ ,  $C_1$ , and  $C_2$  as a function of the elements of  $A$ , which I denote  $a_{ij}$  for the  $i = 1, \dots, 3$  rows and  $j = 1, \dots, 3$  columns of  $A$ :

$$w_f = \frac{-a_{13}a_{22} + a_{12}a_{23}}{a_{11}(-a_{13}a_{22} + a_{12}a_{23}) + a_{13}(a_{23}a_{32} - a_{22}a_{33})} \quad (15)$$

$$w_b = \frac{a_{13}(a_{13}(a_{22}a_{31} - a_{21}a_{32}) + a_{12}(-a_{23}a_{31} + a_{21}a_{33}) + a_{11}(a_{23}a_{32} - a_{22}a_{33}))}{a_{11}(-a_{13}a_{22} + a_{12}a_{23}) + a_{13}(a_{23}a_{32} - a_{22}a_{33})} \quad (16)$$

$$w_y = \frac{-a_{12}^2 a_{23} + a_{13}^2 a_{32} + a_{12}a_{13}(a_{22} - a_{33})}{a_{11}(-a_{13}a_{22} + a_{12}a_{23}) + a_{13}(a_{23}a_{32} - a_{22}a_{33})} \quad (17)$$

$$\beta_f = \frac{-a_{11}a_{13}a_{21} + a_{11}^2 a_{23} + (-1 + a_{13})(a_{23}a_{31} - a_{21}a_{33})}{\Xi} \quad (18)$$

$$\beta_b = \frac{-(a_{13}(-a_{22}a_{31} + a_{21}a_{32}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{11}(-a_{23}a_{32} + a_{22}a_{33})) \times ((-1 + a_{13})a_{21} - a_{11}a_{23})}{\Xi} \quad (19)$$

$$\beta_r = \frac{a_{13}a_{21}^2 - a_{23}(a_{23}a_{31} + a_{21}(a_{11} - a_{33}))}{\Xi} \quad (20)$$

$$\gamma_\pi = \frac{-a_{22}a_{31} + a_{21}a_{32}}{\Omega} \quad (21)$$

$$\gamma_y = \frac{a_{12}a_{31} - a_{11}a_{32}}{\Omega} \quad (22)$$

$$\rho = \frac{a_{23}(a_{12}a_{31} - a_{11}a_{32}) + a_{13}(-a_{22}a_{31} + a_{21}a_{32})}{-a_{12}a_{21} + a_{11}a_{22}} + a_{33}, \quad (23)$$

where

$$\begin{aligned}
\Omega &\equiv a_{13}(-a_{22}a_{31} + a_{21}a_{32}) + a_{12}(a_{23}a_{31} - a_{21}(-1 + a_{33})) \\
&\quad + a_{11}(-a_{23}a_{32} + a_{22}(-1 + a_{33})) \\
\Xi &\equiv a_{12}(a_{13}a_{21}^2 - a_{23}(a_{23}a_{31} + a_{21}(a_{11} - a_{33}))) + a_{11}^2 a_{23}(a_{22} + a_{33}) \\
&\quad + (-1 + a_{13})(a_{13}a_{21}a_{31} + a_{22}(a_{23}a_{31} - a_{21}a_{33})) \\
&\quad - a_{11}(-a_{21}a_{33} + a_{13}(a_{23}a_{31} + a_{21}(a_{22} + a_{33}))).
\end{aligned}$$

Hence, given estimates  $\hat{A}$  and  $\hat{\Sigma}_u$  based on the VAR(1) model

$$z_t = Az_{t-1} + u_t, \quad (24)$$

where  $E(u_t u_t') = \Sigma_u$ , the system (15) - (23) together with (14) identifies the estimates

$$\hat{\theta}(\hat{A}) = (\hat{\omega}_f, \hat{\omega}_b, \hat{\omega}_y, \hat{\beta}_f, \hat{\beta}_b, \hat{\beta}_r, \hat{\gamma}_\pi, \hat{\rho})' \quad (25)$$

and

$$\hat{\Sigma}_\epsilon(\hat{A}, \hat{\Sigma}_u) = \hat{B}^{-1} \hat{\Sigma}_u (\hat{B}^{-1})'. \quad (26)$$

In my Monte Carlo experiments I will use these identifying restrictions to construct structural impulse responses based on OLS estimates of  $A$  for illustrative purposes.

## 4 Theoretic Predictions

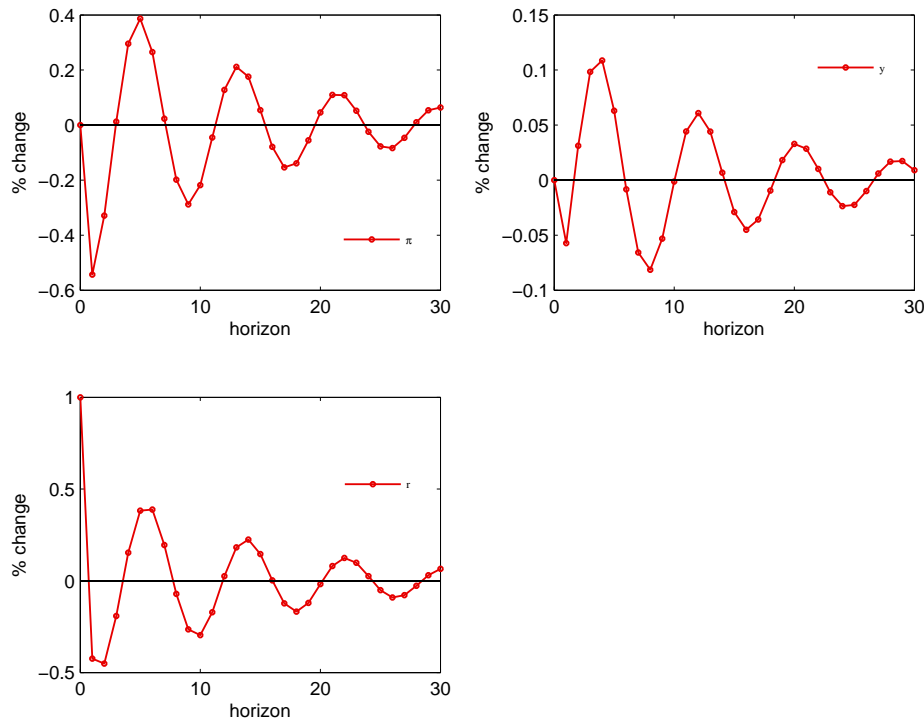
In order to set up a test to disentangle the “forward looking” from the “hybrid” specification I investigate the theoretic predictions of the model outlined in the previous section. Following Linde (2005), I use a benchmark calibration as depicted in table 1, which corresponds to the hybrid specification of the Phillips curve. Using the

Table 1: Benchmark Calibration: The Hybrid Model

$\omega_f$	$\omega_b$	$\omega_y$	$\beta_f$	$\beta_b$	$\beta_r$	$\gamma_\pi$	$\gamma_y$	$\rho$	$\sigma_\pi$	$\sigma_y$	$\sigma_r$
0.3	0.7	0.13	0.3	0.7	0.09	1.5	0.5	0.5	0.3	0.33	0.431

methods described in Christiano (2002) I numerically solve for the VAR(1) solution (12) and generate theoretic impulse response functions, which are graphically represented in figure 1.

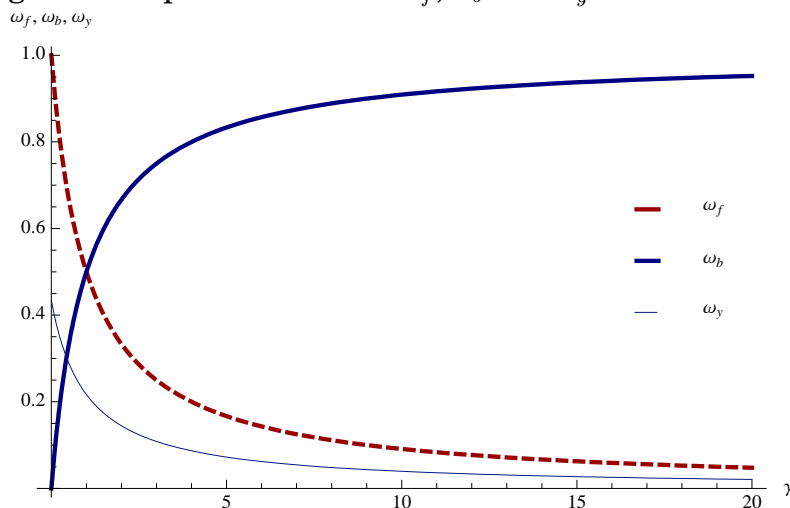
Figure 1: Theoretic IRs for a 1% shock to  $r$  with  $\omega_f \approx 0.3$  (hybrid)



To construct the alternative specification of the fully forward looking model I decompose the calibration in table 1 into the deep parameters in equation (6) as follows: I fix  $\beta = 0.99$  and  $\eta = 0.7$  which implies that  $\gamma \approx 2.3$  and  $\xi = 39$  are the values consistent with the benchmark calibration as suggested by Linde (2005). A value of  $\eta = 0.7$  roughly corresponds to the degree of price stickiness as estimated in Smets

& Wouters (2003) and Christiano et al. (2005). Figure 2 depicts the implied values for  $\omega_f$ ,  $\omega_b$  and  $\omega_y$  as a function of  $\gamma$ .

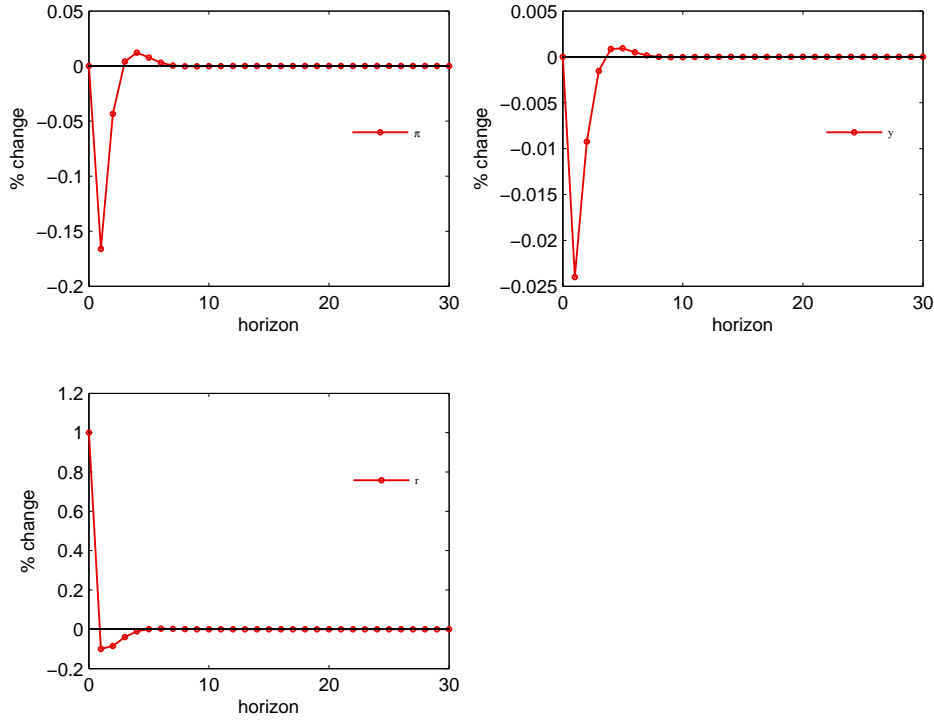
Figure 2: Implied Values for  $\omega_f$ ,  $\omega_b$  and  $\omega_y$  as a function of  $\gamma$



The extreme case of a “fully forward looking” model corresponds to the case where the inflation indexation parameter  $\gamma$  tends to zero, which in turn implies  $\omega_f \approx 1$ ,  $\omega_b \approx 0$ , and  $\omega_y \approx 0.433$ . The theoretic impulse response functions of the fully forward looking model are depicted in figure 3.

To construct a test based on impulse response functions I need to identify a null hypothesis which reflects the distinguishing features between figures 1 and 3. Since the fully forward looking model does not have any internal propagation mechanisms it is reasonable to believe that the dynamic paths of  $y_t$  and  $\pi_t$  in response to a monetary shock are identical, which is confirmed by the impulse response functions plotted in figure 3, where the impulse response of inflation obviously is just a slightly scaled version of the impulse response of the output gap. Based on this simple observation the next section develops a Wald test, based on the estimated impulse response functions for the three equation system (1) – (3).

Figure 3: Theoretic IRs for a 1% shock to  $r$  with  $\omega_f \approx 1$  (hybrid)



## 5 An Impulse Response Based Test

This section develops an impulse response based Wald test in order to disentangle a fully forward looking model as depicted in figure 3 from the hybrid alternative illustrated in figure 1. As shown in section 3 for the given model, an OLS estimate of the VAR(1) coefficient matrix,  $\hat{A}$ , is sufficient to fully identify all the parameters of the system (1) – (3) and hence I can compute estimated structural impulse responses for any forecast horizon  $j$  given by

$$\hat{\Phi}_j = \hat{A}^j \hat{B}. \quad (27)$$

Furthermore, an impulse response function (or path) with forecast horizon  $h$  is then defined by the  $3(h + 1) \times 3$  matrix

$$\hat{\Phi}(1, h) = \begin{bmatrix} I_3 \\ \hat{\Phi}_1 \\ \vdots \\ \hat{\Phi}_h \end{bmatrix}.$$

Jordà (2007) shows that the joint asymptotic distribution of the vector  $\hat{\phi}(1, h) \equiv \text{vec}(\hat{\Phi}(1, h))$  is normal with variance covariance matrix  $\hat{\Omega}_\Phi$ , whose derivation is shown in Jordà (2007). Based on this result, I can set up a Wald test for the null hypothesis  $H_0 : \hat{\phi}_\pi(1, h) - \hat{\phi}_y(1, h) = 0$ , as motivated in the previous section, and the alternative  $H_a : \hat{\phi}_\pi(1, h) - \hat{\phi}_y(1, h) \neq 0$  by computing the Wald statistic

$$W = (R\hat{\phi}(1, h) - b)'(R\hat{\Omega}_\Phi R')^{-1}(R\hat{\phi}(1, h) - b) \sim \chi_h^2, \quad (28)$$

where  $R = [0 \ 0 \ 1] \otimes I_h \otimes [1 \ -1 \ 0]$ ,  $\hat{\phi}_\pi(1, h)$  and  $\hat{\phi}_y(1, h)$  correspond to the impulse response functions of output and inflation in response to a 1% shock to interest rates, respectively, and  $b = 0$  is a vector of zeros.

To illustrate the properties of the test constructed above I run Monte Carlo simulations using the following steps:

1. Define a discrete grid  $G \equiv \{g_1, g_2, \dots, g_J\} \subset [0, \infty)$ , where  $J \in \mathbb{N} \setminus 0$  is the cardinality of  $G$ .
2. Pick  $\tilde{\gamma} = g_j$ .
3. Solve the system (1) – (3) using the methods described in Christiano (2002) with the calibration in table 1 and  $\omega_f(\tilde{\gamma})$ ,  $\omega_b(\tilde{\gamma})$ , and  $\omega_y(\tilde{\gamma})$ .

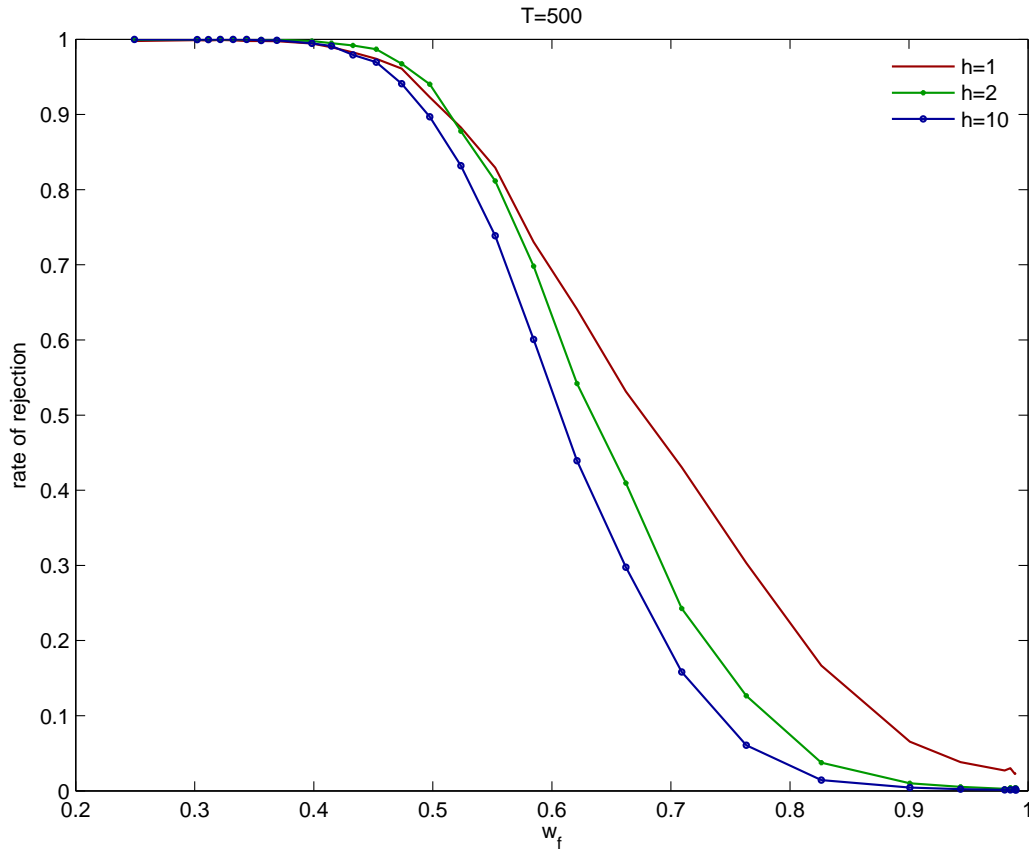
4. Generate  $N$  samples with  $T$  observations from the solution (12).
5. Estimate the VAR(1) coefficient matrix  $A$  in (24) for each sample.
6. Construct estimated impulse response functions  $\hat{\Phi}_h(\hat{A})$  for each sample
7. Test  $H_0$  using (28) at confidence level  $1 - \alpha$ .
8. Report average rejection rate of  $H_0$ .
9. Repeat steps 2 – 8 for each  $j \in \{1, \dots, J\}$ .

It is worth mentioning at this point that impulse response functions for the system (1) – (3) do not exist for values of  $\gamma \in [3, 18.5]$ , which corresponds to parameter values of  $\omega_f \in [0.051, 0.249]$  and  $\omega_b \in [.756, .958]$ , consistent with my decomposition described in section 4. The reason for this is that the only stable solution of the system (1) – (3) for the parameter ranges mentioned above is  $A \approx 0$  and hence one cannot compute impulse response functions. Since the estimates by Linde (2005) as well as Gali et al. (2005) lie in the stable region my test is well defined for the relevant subset of the parameter space.

Consequently, I run Monte Carlo experiments for a grid  $G \subset (0, 3]$  computing  $N = 5000$  samples of  $T \in \{200, 500\}$  observations and forecast horizons  $h \in \{1, 2, 10\}$  to illustrate the properties of my test at a confidence level of  $1 - \alpha = 0.95$ . Figures 4 and 5 as well as tables 3 and 2 summarize the results of the Monte Carlo experiments defined above.

Not too surprisingly figure 4 highlights that the power of my test significantly decreases as the forecast horizon,  $h$ , increases, however, in either case the test clearly rejects the null hypothesis,  $H_0$ , for the calibration corresponding to the estimates of Linde (2005).

Figure 4: Power of the 95% confidence Wald test under  $H_0 : \hat{\phi}_\pi(1, h) = \hat{\phi}_y(1, h)$  vs.  $H_a : \hat{\phi}_\pi(1, h) \neq \hat{\phi}_y(1, h)$  for  $N = 5000$  simulations of  $T = 500$  observations



The Monte Carlo experiment for a sample size of  $T = 200$  clearly illustrates that the test, which is based on the joint asymptotic distribution of the impulse response functions, quickly loses power once the sample size gets too small. A sample of  $T = 200$ , however, is still a reasonable size for macro data, and the test still has sufficient power to reject the “hybrid” model suggested by Linde (2005).

**Figure 5: Power of the 95% confidence Wald test under  $H_0 : \hat{\phi}_\pi(1, h) = \hat{\phi}_y(1, h)$  vs.  $H_a : \hat{\phi}_\pi(1, h) \neq \hat{\phi}_y(1, h)$ , for  $N = 5000$  simulations of  $T = 200$  observations**

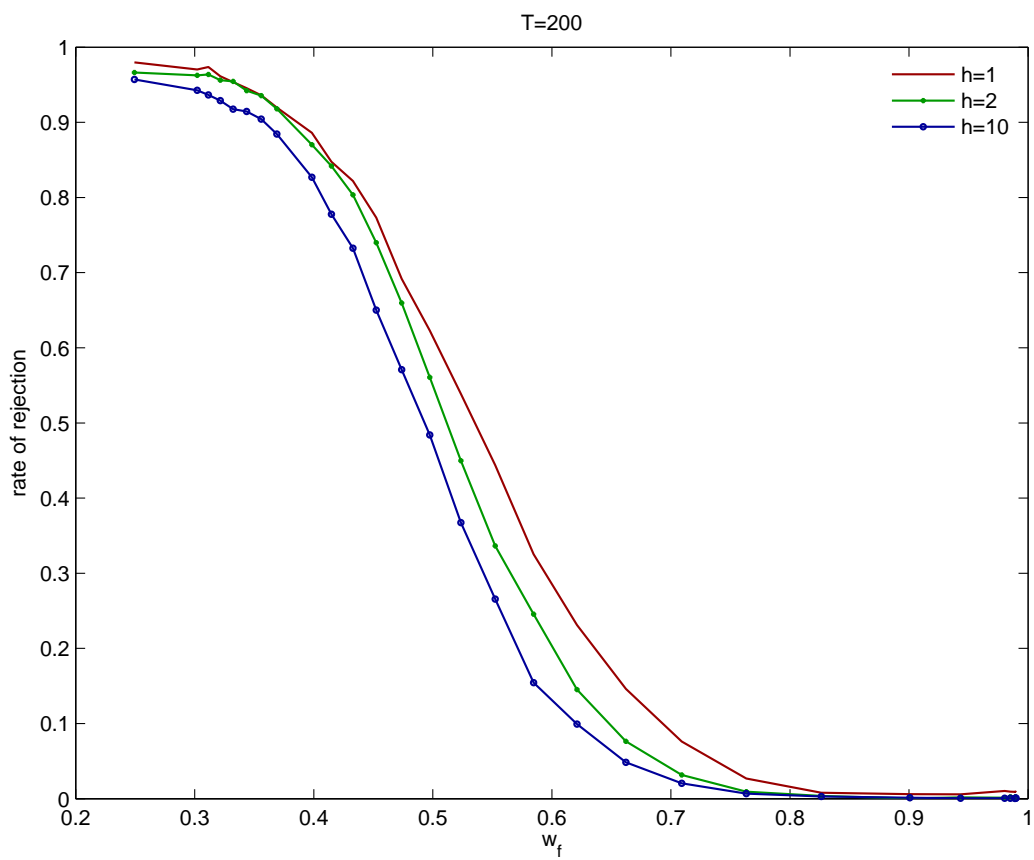


Table 2: Monte Carlo Evaluation of the Wald Test:  $T = 500$ ,  $N = 5000$  and  $\alpha = 0.05$

$w_f$	$w_b$	$h = 1$	$h = 2$	$h = 10$
0.9899	0.0001	0.0238	0.0038	0.0010
0.9895	0.0005	0.0232	0.0026	0.0022
0.9890	0.0010	0.0224	0.0038	0.0016
0.9851	0.0050	0.0302	0.0042	0.0012
0.9803	0.0099	0.0272	0.0028	0.0012
0.9433	0.0476	0.0382	0.0054	0.0022
0.9008	0.0910	0.0654	0.0100	0.0044
0.8264	0.1669	0.1668	0.0374	0.0142
0.7633	0.2313	0.3032	0.1264	0.0606
0.7092	0.2865	0.4306	0.2426	0.1582
0.6622	0.3344	0.5314	0.4094	0.2976
0.6211	0.3764	0.6408	0.5420	0.4394
0.5848	0.4135	0.7304	0.6982	0.6006
0.5525	0.4464	0.8292	0.8116	0.7386
0.5235	0.4759	0.8828	0.8778	0.8318
0.4975	0.5025	0.9230	0.9402	0.8970
0.4739	0.5266	0.9610	0.9674	0.9410
0.4525	0.5484	0.9742	0.9868	0.9696
0.4329	0.5684	0.9826	0.9918	0.9792
0.4149	0.5868	0.9894	0.9948	0.9912
0.3984	0.6036	0.9944	0.9978	0.9946
0.3690	0.6336	0.9974	0.9994	0.9988
0.3559	0.6470	0.9978	1.0000	0.9984
0.3436	0.6595	0.9978	0.9998	0.9998
0.3322	0.6711	0.9988	0.9998	0.9998
0.3215	0.6820	0.9992	0.9998	0.9998
0.3115	0.6923	0.9984	0.9998	0.9996
0.3021	0.7019	0.9990	0.9998	0.9996
0.2494	0.7557	0.9978	0.9992	0.9998

\* The p-values reported correspond to the average rate of rejection over the 5000 simulations at an  $\alpha = 0.05$  confidence level.

Table 3: Monte Carlo Evaluation of the Wald Test:  $T = 200$ ,  $N = 5000$  and  $\alpha = 0.05$

$w_f$	$w_b$	$h = 1$	$h = 2$	$h = 10$
0.9899	0.0001	0.0084	0.0020	0.0008
0.9895	0.0005	0.0096	0.0022	0.0006
0.9890	0.0010	0.0092	0.0018	0.0004
0.9851	0.0050	0.0094	0.0026	0.0006
0.9803	0.0099	0.0102	0.0012	0.0006
0.9433	0.0476	0.0058	0.0018	0.0006
0.9008	0.0910	0.0060	0.0004	0.0012
0.8264	0.1669	0.0080	0.0036	0.0030
0.7633	0.2313	0.0268	0.0094	0.0068
0.7092	0.2865	0.0760	0.0316	0.0204
0.6622	0.3344	0.1462	0.0764	0.0484
0.6211	0.3764	0.2310	0.1452	0.0992
0.5848	0.4135	0.3252	0.2454	0.1544
0.5525	0.4464	0.4438	0.3364	0.2656
0.5235	0.4759	0.5388	0.4498	0.3674
0.4975	0.5025	0.6230	0.5608	0.4840
0.4739	0.5266	0.6918	0.6596	0.5708
0.4525	0.5484	0.7732	0.7398	0.6502
0.4329	0.5684	0.8220	0.8034	0.7326
0.4149	0.5868	0.8476	0.8418	0.7776
0.3984	0.6036	0.8860	0.8702	0.8270
0.3690	0.6336	0.9196	0.9180	0.8844
0.3559	0.6470	0.9360	0.9354	0.9044
0.3436	0.6595	0.9454	0.9422	0.9144
0.3322	0.6711	0.9534	0.9544	0.9176
0.3215	0.6820	0.9614	0.9560	0.9288
0.3115	0.6923	0.9736	0.9636	0.9364
0.3021	0.7019	0.9702	0.9624	0.9426
0.2494	0.7557	0.9798	0.9662	0.9570

\* The p-values reported correspond to the average rate of rejection over the 5000 simulations at an  $\alpha = 0.05$  confidence level.

## 6 Concluding Remarks

The test developed in the previous sections is a tool to empirically evaluate theoretic models for the dynamics of inflation solely based on the shape of the implied impulse responses to exogenous shocks. This is particularly useful when one wants to be agnostic about the precise econometric specification and wants to evaluate theoretic predictions without imposing restrictions on the dynamics implied by the data. This paper develops and evaluates the test with the use of a simple theoretic model which has a VAR(1) solution and all the structural parameters are fully identified. Given that knowledge one can use OLS to estimate the reduced form as a VAR(1) on simulated data as illustrated in the previous section. In practice, however, one does not know the structure of the underlying DGP and hence a convenient way to estimate “model free” impulse response functions is the local projection method suggested by Jordà (2005). Together with the joint asymptotic distribution of these impulse responses generated by local projections, as derived in Jordà (2007), one can apply the test in the exact same way as demonstrated in the previous sections. While this test does not answer the question of how to estimate the structural parameters of the preferred model it is a useful tool for empirically evaluating opposing theories without having to specify an explicit econometric model.

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